AP

AP® Physics C: Mechanics 2014 Free-Response Questions

CollegeBoard

TABLE OF INFORMATION, EFFECTIVE 2012

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant, $G = 6.67 \times 10^{-11}$ m^3/kg•s^2
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8$ m/s^2
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\text{ J•s} = 4.14 \times 10^{-15}\text{ eV•s}$
	$hc = 1.99 \times 10^{-25}\text{ J•m} = 1.24 \times 10^{3}\text{ eV•nm}$
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12}\text{ C}^2/\text{N•m}^2$
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^{9}\text{ N•m}^2/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\text{ (T•m)/A}$
Magnetic constant,	$k' = \mu_0/4\pi = 1 \times 10^{-7}\text{ (T•m)/A}$
1 atmosphere pressure,	$1\text{ atm} = 1.0 \times 10^{5}\text{ N/m}^2 = 1.0 \times 10^{5}\text{ Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron-volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

ADVANCED PLACEMENT PHYSICS C EQUATIONS, EFFECTIVE 2012

MECHANICS

$$\upsilon = \upsilon_0 + at$$

$$x = x_0 + \upsilon_0 t + \frac{1}{2}at^2$$

$$\upsilon^2 = \upsilon_0^2 + 2a(x - x_0)$$

$$\Sigma \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$$

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

$$\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$$

$$\mathbf{p} = m\mathbf{v}$$

$$F_{fric} \le \mu N$$

$$W = \int \mathbf{F} \cdot d\mathbf{r}$$

$$K = \frac{1}{2}m\upsilon^2$$

$$P = \frac{dW}{dt}$$

$$P = \mathbf{F} \cdot \mathbf{v}$$

$$\Delta U_g = mgh$$

$$a_c = \frac{\upsilon^2}{r} = \omega^2 r$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$\Sigma \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$$

$$I = \int r^2 dm = \Sigma m r^2$$

$$\mathbf{r}_{cm} = \Sigma m\mathbf{r} / \Sigma m$$

$$\upsilon = r\omega$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$$

$$K = \frac{1}{2}I\omega^2$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

a = acceleration
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
N = normal force
P = power
p = momentum
r = radius or distance
$\mathbf{r}$ = position vector
T = period
t = time
U = potential energy
υ = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$$\mathbf{F}_s = -k\mathbf{x}$$

$$U_s = \frac{1}{2}kx^2$$

$$x = x_{\max}\cos(\omega t + \phi)$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$$

$$U_G = -\frac{Gm_1m_2}{r}$$

ELECTRICITY AND MAGNETISM

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

$$E = -\frac{dV}{dr}$$

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

$$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

$$C = \frac{Q}{V}$$

$$C = \frac{\kappa\epsilon_0 A}{d}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$I = \frac{dQ}{dt}$$

$$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$$

$$R = \frac{\rho\ell}{A}$$

$$\mathbf{E} = \rho\mathbf{J}$$

$$I = Nev_d A$$

$$V = IR$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$P = IV$$

$$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential or stored energy
V = electric potential
υ = velocity or speed
ρ = resistivity
ϕ_m = magnetic flux
κ = dielectric constant

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$$

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\boldsymbol{\ell} \times \mathbf{r}}{r^3}$$

$$\mathbf{F} = \int I\,d\boldsymbol{\ell} \times \mathbf{B}$$

$$B_s = \mu_0 n I$$

$$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$$

$$\mathcal{E} = \oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d\phi_m}{dt}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$U_L = \frac{1}{2}LI^2$$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

Rectangular Solid

$V = \ell w h$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r \ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

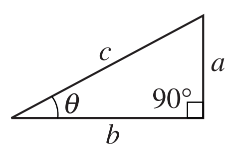
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^x\,dx = e^x$$

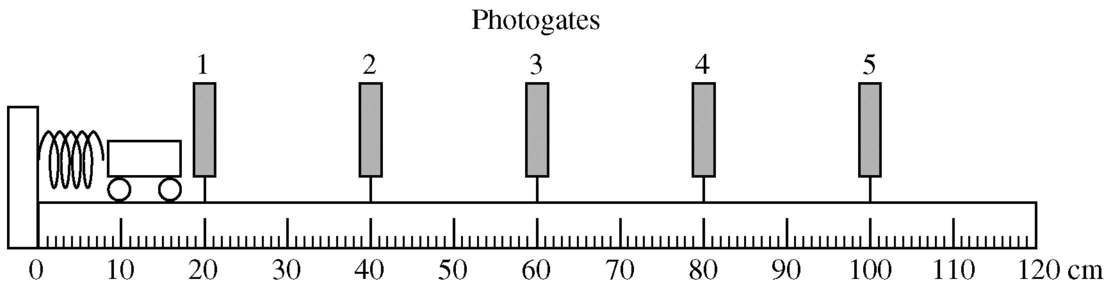
$$\int \frac{dx}{x} = \ln|x|$$

$$\int \cos x\,dx = \sin x$$

$$\int \sin x\,dx = -\cos x$$

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Mech. 1.

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude $F_s = As^2 + Bs$, where s is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

(a) Derive an expression for the potential energy U as a function of the compression s. Express your answer in terms of A, B, s, and fundamental constants, as appropriate.

In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, $A = 200\ \text{N/m}^2$ and $B = 150\ \text{N/m}$. The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart.

(b) Calculate the following:

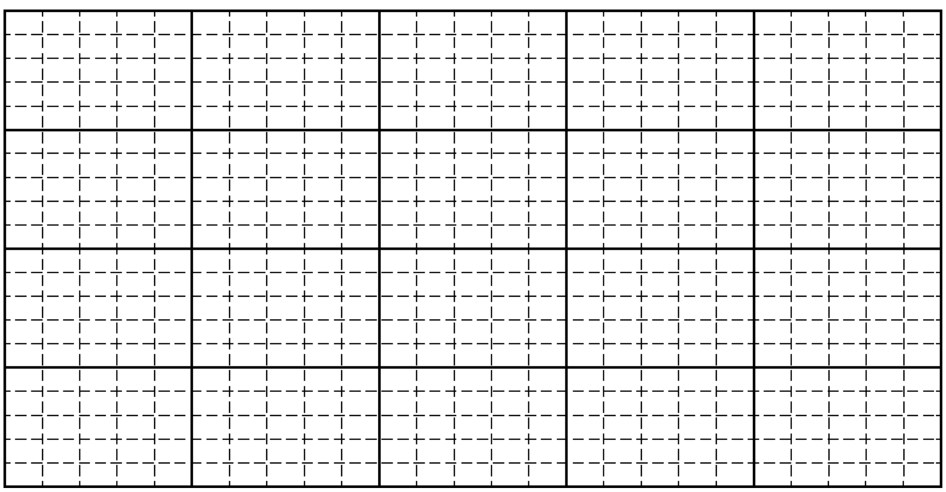
i. The speed of the cart immediately after it loses contact with the spring

ii. The impulse given to the cart by the spring

In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of $0.320\ \text{m/s}$ after the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of $\pm 0.002\ \text{m/s}$. The positions are measured with a precision of ± 0.005 m.

Photogate	1	2	3	4	5
Cart speed (m/s)	0.412	0.407	0.399	0.374	0.338
Photogate position (m)	0.20	0.40	0.60	0.80	1.00

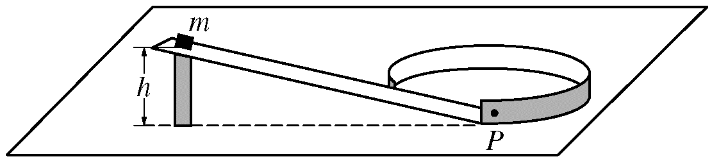
(c) On the axes below, plot the data points for the speed v of the cart as a function of position x. Clearly scale and label all axes, as appropriate.



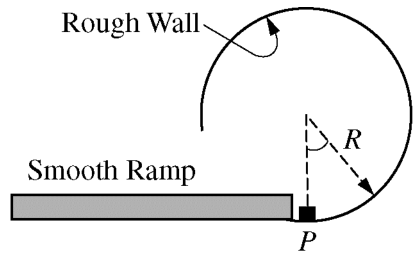
(d)

i. Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference.

ii. From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.



Side View



Top View

Mech. 2.

A small block of mass m starts from rest at the top of a frictionless ramp, which is at a height h above a horizontal tabletop, as shown in the side view above. The block slides down the smooth ramp and reaches point P with a speed v_0. After the block reaches point P at the bottom of the ramp, it slides on the tabletop guided by a circular vertical wall with radius R, as shown in the top view. The tabletop has negligible friction, and the coefficient of kinetic friction between the block and the circular wall is μ.

(a) Derive an expression for the height of the ramp h. Express your answer in terms of v_0, m, and fundamental constants, as appropriate.

A short time after passing point P, the block is in contact with the wall and moves with a speed of v.

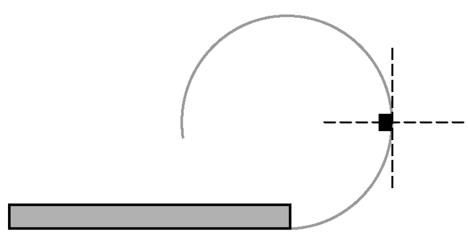
(b)

i. Is the vertical component of the net force on the block upward, downward, or zero?

____ Upward ____ Downward ____ Zero

Justify your answer.

ii. On the figure below, draw an arrow starting on the block to indicate the direction of the horizontal component of the net force on the moving block when it is at the position shown.



Top View

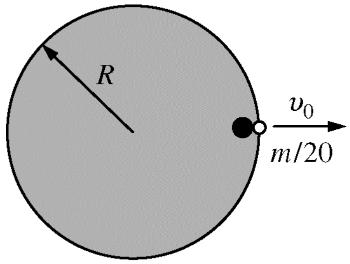
Justify your answer.

Express your answers to the following in terms of v_0, v, m, R, μ, and fundamental constants, as appropriate.

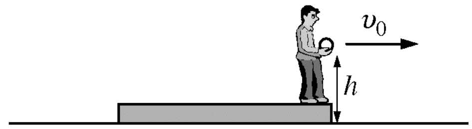
(c) Determine an expression for the magnitude of the normal force N exerted on the block by the circular wall as a function of v.

(d) Derive an expression for the magnitude of the tangential acceleration of the block at the instant the block has attained a speed of v.

(e) Derive an expression for $v(t)$, the speed of the block as a function of time t after passing point P on the track.



Top View



Side View

Mech. 3.

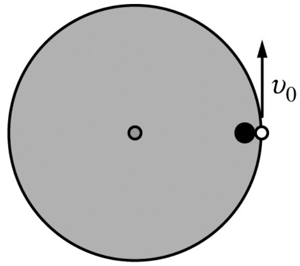
A large circular disk of mass m and radius R is initially stationary on a horizontal icy surface. A person of mass $m/2$ stands on the edge of the disk. Without slipping on the disk, the person throws a large stone of mass $m/20$ horizontally at initial speed v_0 from a height h above the ice in a radial direction, as shown in the figures above. The coefficient of friction between the disk and the ice is μ. All velocities are measured relative to the ground. The time it takes to throw the stone is negligible. Express all algebraic answers in terms of m, R, v_0, h, μ, and fundamental constants, as appropriate.

(a) Derive an expression for the length of time it will take the stone to strike the ice.

(b) Assuming that the disk is free to slide on the ice, derive an expression for the speed of the disk and person immediately after the stone is thrown.

(c) Derive an expression for the time it will take the disk to stop sliding.

GO ON TO THE NEXT PAGE.



Top View

The person now stands on a similar disk of mass m and radius R that has a fixed pole through its center so that it can only rotate on the ice. The person throws the same stone horizontally in a tangential direction at initial speed v_0, as shown in the figure above. The rotational inertia of the disk is $mR^2/2$.

(d) Derive an expression for the angular speed ω of the disk immediately after the stone is thrown.

(e) The person now stands on the disk at rest $R/2$ from the center of the disk. The person now throws the stone horizontally with a speed v_0 in the same direction as in part (d). Is the angular speed of the disk immediately after throwing the stone from this new position greater than, less than, or equal to the angular speed found in part (d) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

STOP

END OF EXAM